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Asset Correlations for Credit Card Defaults

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Abstract

The capital requirements formula within the Basel II Accord is based on a Merton one factor model and in the case of credit cards an asset correlation of 4% is assumed. In this paper we estimate the asset correlation for two datasets assuming the one factor model. We find that the asset correlations assumed by Basel II are much higher than those observed in the datasets we analyse. We show the reduction in capital requirements that a typical lender would have if the values we estimated were implemented in the Basel Accord in place of the current values.

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1. Introduction

The aim of this paper is to present estimates of asset correlations for credit card portfolios. The recent difficulties of banks throughout the world has focussed attention on the amount and detailed nature of regulations that the banking sector operates under. Large banks in the G10 countries are obliged by their regulators to follow the Basel II Accord (Basel Committee on Banking Supervision: 2006) when deciding on the amount of capital to hold to protect their shareholders against operational, market and credit risk. Under the Accord banks may follow a standardised approach or an IRB approach to calculating their risk capital. Those following the latter, probably the majority of large banks, must calculate their capital requirements for different loan segments using an equation based on the Vasicek formula (Vasicek 1987) with different assumed asset correlation parameters for different types of loan portfolio. For many retail segments, such as mortgages and qualifying revolving retail exposures, the Accord assumes an asset correlation of a specific value: 15% for residential mortgages and 4% for revolving exposures. For other types of retail loans, and for other segments, for example corporate loans and loans to small and medium sized enterprises, the asset correlation depends on the long run probability of default for that portfolio, (as well as maturity for non retail exposures).

However the empirical work on which the assumed asset correlation values for mortgages and revolving credit portfolios were based has not been published and an embryonic literature, mainly examining corporate loans and loans to banks, finds asset correlation values, generally, to be much lower than the values implied by the Basel formula. For example Rösch (2003) estimated corporate asset correlations in Germany by modelling the number of bankruptcies over time. He found values around 0.86% when macroeconomic variables were omitted and 0.52% when they were included. Hamerle et al (2003a) found comparable values for many industries in the G7 countries. In other work Rösch (2005) found that corporate asset correlations varied according to rating class, where the class was allocated by an external ratings agency. In this work Rösch did not include borrower specific effects, however in Hamerle and Rösch (2006) this was attempted for a sample of German firms in

manufacturing and in commerce. The estimates were even smaller, declining from 0.6% to 0.04% when company specific variables were included. In contrast McNeill and Wendin (2007) find within sector asset correlations for a sample of US corporate loans to be around 10.9%. But asset correlations for corporate loans tell us very little indeed about those for consumer loans. Estimates of asset correlations for *consumer loans* or for mortgages are rare. Rösch and Scheule (2004) considered three exposure classes, residential loans, credit cards and other consumer loans from US commercial banks. When, following the Basel Accord, they assume the default probabilities are constant over time, asset correlations of 1.0% for credit cards and 0.98% for real estate loans were estimated. When macroeconomic variables were included, the asset correlations fell to 0.66% and 0.28% respectively.

However there is no published work in which the asset correlations for consumer loan portfolios are estimated and where individual specific effects are included, and there is no published work that reports asset correlations for UK credit card portfolios. This is the aim of this paper. In principle one would expect that the more covariates that are used to explain the probability of default by a borrower the lower the computed asset correlation and empirical evidence for other sectors appear to confirm this. We use two datasets, one relating to individual borrowers and the second relates to all credit cards issued in the UK. We find asset correlation values that are very considerably below the value assumed in the Accord and that the correlation varies systematically with the riskiness of a borrower segment.

The structure of this paper is as follows. In the following section we specify the model and estimation strategy that we use. In section three we present our results and in section four we discuss some implications of them. Section five concludes.

Notational Conventions

Throughout the paper we adopt the following conventions. We assume time is measured in discrete intervals. Let \mathbf{w}_i denote a vector of variables whose observed values are specific to individual i ($i = 1 \dots N$), but that do not vary over time. Let \mathbf{x}_{it} denote a vector of variables whose observed values vary between individuals and

between time periods ($t = 1 \dots T$). Let \mathbf{z}_t denote a vector of variables whose observed values vary between time periods, but not between individuals. Let Y_t denote a variable whose values vary over time but which are not observable. We denote individual variables in upper case and their realisations in lower case. We use l to denote an arbitrary lag. We write $P(\bullet | Y_t = y_t)$ as $p(y_t)$ for convenience.

2. The Model

The model follows Hamerle et al (2003b), Hamerle and Rösch (2006) and Rösch (2005). Consider the following to apply within a particular risk segment. Let A_{it} denote a borrower's assets and a_{it-1} denote the realisation of his asset's in a previous period. These may be in natural logs. Let the return on his assets be denoted R_{it} , so $R_{it} = A_{it} - a_{it-1}$. We assume R_{it} is an unobserved latent variable and is linearly related to its mean, μ_{it} , an unobserved random time specific effect, Y_t , and a random component, ε_{it} . Thus we can write

$$R_{it} = \mu_{it} + bY_t + c\varepsilon_{it}, \quad (1)$$

where R_{it} is normally distributed with mean μ_{it} and standard deviation σ ; $Y_t \sim N(0,1)$ and $\varepsilon_{it} \sim N(0,1)$. We assume Y_t and ε_{it} are independent and Y_t is serially uncorrelated. The variance of bY_t is b^2 and that of $c\varepsilon_{it}$ is c^2 and so, given independence of Y_t and ε_{it} , $\sigma^2 = b^2 + c^2$. One implication of Equation 1 is that conditional on Y_t values of R for any two cases, i and j , are independent. Another implication is that since R_{it} is unobserved we cannot estimate c .

Equation 1 may be standardised to give

$$\frac{R_{it} - \mu_{it}}{\sigma} = \frac{b}{\sigma} Y_t + \frac{c}{\sigma} \varepsilon_{it}. \quad (2)$$

Now suppose, following Rösch (2005), Gordy (2003) and Hamerle et al (2003b) we write Equation 2 as

$$\frac{R_{it} - \mu_{it}}{\sigma} = \tilde{b} Y_t + \sqrt{1 - \tilde{b}^2} \varepsilon_{it}, \quad (3)$$

where $\tilde{b} = \frac{b}{\sigma}$. The correlation between the standardised values of R_{it} and R_{jt} is

$$\tilde{b}^2 = \left(\frac{b}{\sigma} \right)^2 \quad (\text{Schonbucher: 2000}).$$

Following Merton (1974) a borrower defaults when the value of his assets falls below a threshold level. If we denote this threshold as k_{it} , the probability of default for borrower i is

$$\begin{aligned} P(A_{it} < k_{it}) &= P\left(\frac{R_{it} - \mu_{it}}{\sigma} < \frac{k_{it} - a_{it-1} - \mu_{it}}{\sigma} \right) \\ &= P\left(\tilde{b} Y_t + \sqrt{1 - \tilde{b}^2} \varepsilon_{it} < \alpha_{it} \right), \end{aligned} \quad (4)$$

where $\alpha_{it} = \frac{k_{it} - a_{it-1} - \mu_{it}}{\sigma}$. Thus the probability of default can be written as

$$P\left(\varepsilon_{it} < \frac{\alpha_{it} - \tilde{b} Y_t}{\sqrt{1 - \tilde{b}^2}} \right), \quad (5)$$

and conditional on the realisation $Y_t = y_t$ this probability equals

$$p(y_t) = \Phi\left(\frac{\alpha_{it} - \tilde{b} y_t}{\sqrt{1 - \tilde{b}^2}} \right), \quad (6)$$

where Φ denotes the standard normal cumulative distribution.

Equation 3 assumes that the standardised return does not vary systematically over time; instead it varies only randomly according to Y_t and ε_{it} . This is the assumption

of Basel 2. But now suppose that the standardised return, and so the probability that case i defaults, is affected by observable states of the economy, possibly lagged. For example if interest rates increase we may expect that an average individual is less able to repay his outstanding loans. If, following Hamerle et al (2003b and 2006), we assume that the mean of the return depends linearly on observable macroeconomic variables, as well as static characteristics of a borrower (\mathbf{w}_i) and time-varying characteristics of the borrower (\mathbf{x}_{it}), we can replace Equation 1 by

$$R_{it} = \beta_0 + \mathbf{w}_i^T \boldsymbol{\beta} + \mathbf{x}_{it}^T \boldsymbol{\gamma} + \mathbf{z}_t^T \boldsymbol{\delta} + bY_t + c\varepsilon_{it}, \quad (7)$$

where β_0 is a constant and $\boldsymbol{\beta}, \boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are vectors of parameters to be estimated. By substitution Equation 6 becomes

$$P(A_{it} < k_{it} \mid \mathbf{w}_i = w_i, \mathbf{x}_{it} = x_{it}, \mathbf{z}_t = z_t, Y_t) = \Phi\left(\frac{\alpha_{it} - w_i^T \tilde{\boldsymbol{\beta}} - x_{it}^T \tilde{\boldsymbol{\gamma}} - z_t^T \tilde{\boldsymbol{\delta}} - \tilde{b}Y_t}{\sqrt{1 - \tilde{b}^2}}\right). \quad (8)$$

where $\tilde{\boldsymbol{\beta}} = \boldsymbol{\beta} / \sigma$, $\tilde{\boldsymbol{\gamma}} = \boldsymbol{\gamma} / \sigma$ and $\tilde{\boldsymbol{\delta}} = \boldsymbol{\delta} / \sigma$.

To parameterise this model we follow Hamerle et al (2006) Equation 8 and also assume α_{it} is a constant. There the log-likelihood is derived by supposing that we observe a particular default pattern across borrowers in period t , and finding an expression that equals the probability of observing that pattern, conditional on the realisation y_t of Y_t . Thus if we observe a default pattern $(d_{it}, d_{2t}, \dots, d_{Nt})$ where $d_{it} = 1$ denotes that borrower i in period t defaults, the log-likelihood is

$$LL = \sum_{t=1}^T \ln \left(\int_{-\infty}^{+\infty} h(y_t) \phi(y_t) dy_t \right), \quad (9)$$

where $h(y_t) = \prod_{i=1}^{N_t} P_{it}(\bullet \mid Y_t = y_t)^{d_{it}} (1 - P_{it}(\bullet \mid Y_t = y_t))^{1-d_{it}}$ and ϕ denotes the standard normal density function.

Essentially equation 8, together with the default condition, is a mixed fixed effects - random effect probit model where the fixed effects are $\mathbf{w}_i^T \tilde{\boldsymbol{\beta}} / \sqrt{1 - \tilde{b}^2}$, $\mathbf{x}_{it}^T \tilde{\boldsymbol{\gamma}} / \sqrt{1 - \tilde{b}^2}$ and $\mathbf{z}_i^T \tilde{\boldsymbol{\delta}} / \sqrt{1 - \tilde{b}^2}$ and the random effect is $(\tilde{b} / \sqrt{1 - \tilde{b}^2}) Y_i$. Notice that unlike more conventional panel models the random effect is over time not over cases. Correspondingly the integration in the maximum likelihood function (Equation 9) is over the random effect that varies over time.

One can parameterise such a model by maximising the LL function where $\frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}}$ is the variance of the random effect and the asset correlation between any two borrowers within the same segment is

$$\rho = \frac{\left(\frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}} \right)^2}{\left(\frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}} \right)^2 + \left(\frac{\sqrt{1 - \tilde{b}^2}}{\sqrt{1 - \tilde{b}^2}} \right)^2} = \tilde{b}^2. \quad (10)$$

The value of $\left(\frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}} \right)$ can be estimated as part of the maximum likelihood estimation and so we can calculate the asset correlation using Equation 10. We parameterised Equations 6 and 8 using maximum likelihood and estimated the standard error of rho using the delta method of Billingsley (1986).

Alternatively one can obtain estimates of the asset correlation in a portfolio using aggregated data instead of data for individual borrowers. Here we closely follow the models of Gordy and Heitfield (2002), Rösch (2003) and Rösch (2005). Define

$$D_t = \sum_{i=1}^{N_t} d_{it} \text{ where } d_{it} \text{ is defined above and } N_t \text{ is the number of active borrowers in}$$

period t . Assume Equation 1 explains returns. By assumption, conditional on the realisation $Y_t = y_t$, the returns of any two borrowers within a segment are independent. Therefore the distribution of the number of defaults in period t , conditional on the realisation of Y_t , is binomial with parameters $(N_t, p(y_t))$ where

$p(y_t)$ is given by Equation 6. The probability of D_t defaults in period t may be written as

$$l(D_t | Y_t) = \binom{N_t}{D_t} p(y_t)^{D_t} (1 - p(y_t))^{N_t - D_t}. \quad (11)$$

By assumption Y_t and ε_{it} are serially independent so the unconditional marginal log-likelihood (which we denote LL) can be found by integrating Equation 11 over all possible values of Y_t , taking the product over time periods $t=1 \dots T$ and taking logs. This gives

$$LL = \sum_{t=1}^T \ln \left(\int_{-\infty}^{+\infty} \binom{N_t}{D_t} p(y_t)^{D_t} (1 - p(y_t))^{N_t - D_t} \phi(y_t) dy_t \right). \quad (12)$$

Alternatively, if we assume returns are explained by Equation 7 rather than by Equation 1, then $p(y_t)$ in Equation 12 may be replaced by $p(y_t, z_t)$, which is Equation 8 with β and γ restricted to a vector of zeros.

For an individual risk segment Equation 12 allows one to parameterise Equation 6 (or Equation 8 with the above restrictions) using merely data on the number of defaults, the number of active accounts and observable time varying covariates. As Rösch (2005) states, if time varying macroeconomic variables are included in Equation 6 the proportion of the variance of R_{it} that is explained by the random effect will be lower than if the macroeconomic variables were omitted.

We parameterised Equation 6 using the NLMIXED procedure in SAS. We used Gaussian adaptive quadrature (Pinheiro and Bates: 1995) to compute the integral over the time varying marginal effects and the delta method of Billingsley (1986) to compute the approximate standard errors for rho. Both are standard procedures in SAS.

3. Results

3.1 Account Level Data

We use two datasets. The first dataset is a randomly selected sample of approximately 200,000 credit card accounts that relate to a single credit card issued by a financial institution. We define default as missing a third due monthly payment or the account being written off, and the time of default is the first month in which either event happens. The observations cover 87 consecutive months from the late 1990s to the mid 2000s. The dataset is unbalanced: accounts were opened at different points in time. Some defaulted in the observation period, others did not. Due to software constraints we took a random sample of 20% of the accounts to estimate the parameters of Equation 6 for the portfolio as a whole. The results are shown in Table 1. These show the estimated asset correlation value of 0.396%. This is considerably below the value of 4.00% given in the June 2006 version of the Basel II formula for revolving accounts not in default.

Table 1 Here

The Basel Accord states that the risk weight formula is to apply to segments of equal risk within a portfolio, ('risk buckets'). In practice risk segments are often identified by borrowers operating in the same country or, in the case of corporate loans, in the same industry. In the case of credit cards one could segment a portfolio according to very many variables, for example income level, occupation, address region, age and so on. To investigate the effect of a segmentation we divided the portfolio into risk groups where each group consisted of borrowers in the same decile of generic risk score. Each decile therefore consisted of approximately 20,000 accounts. The results of estimating Equation 6 for selected deciles is shown in Table 2. The results show a systematic relationship between asset correlation and generic risk score. At higher risk score deciles (lower risk) the asset correlation is smaller than at higher risk score

deciles (higher risk). In decile 10, the asset correlation was not statistically different from zero.

Table 2 Here

Another possibility is that the asset correlations in Basel II relate only to periods when credit portfolios are stressed. To examine this we re-estimated the asset correlations for two separate periods: June 1999 to September 2001 inclusive and October 2001 to November 2003 inclusive. The former is a period when UK economic activity is above the trend for the index of (real) production and the latter a period below trend. We estimated the trend as a 61 month centred moving average. The results are shown in Table 3. We find that asset correlations are lower in recession than when the economy is not in recession. We could have identified shorter periods when the economy was deeper in a recession but that would reduce the number of observations so that the results may not be robust – the estimates are derived from the time series property of the data.

Table 3 Here

Table 4 shows the effect of additional covariates on the estimated value of ρ . We consider two alternative equations; one including a variable that varies over cases and over time: balance divided by credit limit and a equation which omits such a variable. As expected, in both cases when more covariates are added the value of ρ declines. The equations suggest some collinearity because when balance divided by credit limit is included, many of the other variables become insignificant.

Table 4 Here

However a caveat to this section is in order. The account level dataset spans only 87 months. The second dataset spans a longer period.

3.2 Aggregate Data

The second dataset is quarterly time series data relating to the repayment performance of all credit card issuers in the UK and was kindly supplied by APACS. The data period is from 1990 Q2 to 2007 Q4 and so spans a much longer period than the account level data.. This dataset does not show the number of accounts that defaulted in each time period, but instead shows the number of accounts that were overdue by various periods, for example by one month, two months and so on. We also had the number of accounts that were written off in each quarter as well as the number of active accounts. We defined a default event as being when an account became three months overdue. The relationship between these concepts can be written as:

$$D_t - R_t = O_t - O_{t-1} + W_t, \quad (13)$$

where D_t is the number of accounts that became three months overdue in month t ; R_t denotes the number of accounts that were more than 3 months overdue in $t-1$ that repaid sufficient amounts to be just 3 months overdue in period t ; O_t denotes the number of accounts that were three or more months overdue in period t ; and W_t denotes the number of accounts that were written off in period t . The dataset contained information on O_t and W_t and prior research experience by the authors suggests that the number of accounts that are 3 months overdue that repay enough to be just three months overdue is very small and so we assume that $R_t = 0$. If this assumption is untrue then we are modelling $D_t - R_t$, the net change in the number of accounts 3 months overdue between t and $t-1$ rather than D_t . The time series of the number of defaults as a percentage of active accounts is shown in Fig. 1. The data show a discernable upward trend since around May 1997, albeit with some perturbations. Table 5 shows the results of parameterising Equation 6. These suggest that the asset correlation of the portfolio of credit cards made available by UK issuers is around 1.8% when estimated in this way. This is considerably higher than the 0.4-0.6% which estimates based on the borrower level dataset suggested. Both sets of values are considerably lower than the values assumed by the Basel 2 formula. Unfortunately because our data is quarterly there would be too few observations in each of the two time periods we identified in the previous section as being periods of recession or growth to estimate robust values of ρ for each period separately.

Fig.1 Here

Table 5 Here

4. Implications

Considering both sets of results the values of asset correlations that we have estimated are similar to those found by other researchers. For example, values around 1% for credit cards in the US were found by Rösch and Scheule (2004). Hamerele et al (2006) found values around 0.6% for manufacturing and 0.1% for Commerce in Germany in the 1990s. However Rösch (2005) is an exception that finds asset correlations for US corporates around 5% when modelling defaults for three separate risk grades, BB, B and CCC. We also found that higher risk segments had higher asset correlations. This was not found by Rösch (op cit) who found that whilst the CCC grade had the highest correlation the lowest was for the B grade. The reduction in the value of rho when other covariates are included in the model is expected and consistent with previous studies.

The Basle II Accord gives the capital requirement, K , for a risk segment of a portfolio of qualifying revolving retail exposures as

$$K = \Phi \left(\frac{\Phi^{-1}(PD) + \rho \Phi^{-1}(0.999)}{\sqrt{1 - \rho}} \right), \quad (14)$$

where PD is the mean probability of default. Table 6 shows the implications for capital requirements for differing PDs and asset correlations. It can be seen that if the mean PD is around 3% and the asset correlations of 0.4% or even 0.6% were applied, the capital requirement, as a factor of LGD would be approximately 25% of that required by the Accord with its given asset correlation of 4%. The over capitalisation is larger if the mean default rate is lower. For example if the mean default rate was 1% the capital requirement would be around 23% of the value required under the Accord.

Table 6 Here

Our finding that higher asset correlations occur in higher risk bands might be concerning to lenders because it is the higher risk band borrowers who are most likely to default. Nevertheless the asset correlations, even in the highest risk band in our data, were not as large as the assumed correlation in the Accord, and it is the assumptions of the Accord that are currently enforced by regulators.

A further implication is that Equation 14 is effectively Equation 6 with α_{it} assumed constant for the segment and the value of Y_t set at $\Phi^{-1}(0.999)$, its extreme expected value on 0.1% of occasions. With assumed values of ρ and $p(y_t)$ one can estimate a mean PD from this equation. This mean PD is unaffected by changes in Y_t because the value of Y_t has been fixed. The estimated value of PD is therefore a through the cycle estimate and may yield an estimate for portfolios with very few observed defaults. Pluto and Tasche (2009) consider a method which uses Equation 6 with an assumed value of ρ to predict upper bounds on PD when there are few defaults in each of three risk grades. If one wishes to use this method for credit card loans and if one uses a value of ρ of 4% then the predicted PDs may be considerably in error.

5. Conclusions

Although the Basel II Accord requires banks who wish to follow an IRB approach to estimating their capital requirements to use an asset correlation for credit card loans of 4%, our results suggest that this is considerably larger than the asset correlations that occur in practice. Our results suggest a value of around half of the required amount. This finding adds to those of other studies which have concentrated on corporate loans so that this conclusion applies to both corporate and consumer loans. We have found that asset correlations are greater for riskier credit card borrowers and that there may be noticeable differences between the correlations of different lenders. We also

find that in periods of stress asset correlations for credit cards are lower than in other periods.

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Table 1
Borrower Level Dataset Parameterisations of Equation 6

	Const	$\text{sd} \left(\frac{\tilde{b}}{\sqrt{1-\tilde{b}^2}} . Y_t \right)$	ρ
=====			
Coeff	na**	0.0630**	0.00396**
SE		(0.0093)	(0.0012)
Chi Sq(1)			45.35

No Groups = 87

Av no of obs per group 19,879

Na = not available for confidentiality reasons.

* denotes significance at 5%; ** denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution.

Ref: \stata\p3p11.log

Table 2
Parameterisations of Equation 6 for Score deciles

		Const	$\text{sd} \left(\frac{\tilde{b}}{\sqrt{1-\tilde{b}^2}} . Y_t \right)$	ρ	Av no of obs per group (N x T)
Decile 1	Coeff	-2.575**	0.0803**	0.0064**	8,089
	SE	(0.0113)	(0.0108)	(0.0017)	
	Chi Sq(1)			67.59	
Decile 3	Coeff	-2.756**	0.0590**	0.0035**	9,695
	SE	(0.0097)	(0.0097)	(0.0011)	
	ChiSq(1)			25.64	
Decile 5	Coeff	-2.902**	0.0506**	0.0026**	10,759
	SE	(0.0098)	(0.0117)	(0.0012)	
	Chi Sq(1)			9.03	
Decile 7	Coeff	-3.024**	0.0466**	0.0022	11,238
	SE	(0.0106)	(0.0148)	(0.0014)	
	Chi Sq(1)			3.88	
Decile 9	Coeff	-3.333**	0.0334	0.0011	9,598
	SE	(0.0165)	(0.0011)	(0.0029)	
	Chi Sq(1)			0.16	
No groups=87.					

* denotes significance at 5%; ** denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution

Ref: stata\p3p14.log, stata\p3p15.log, stata\p3p16.log, stata\p3p19.log

Table 3

Borrower Level Parameterisation for Different States of the Macroeconomy

	Const		$\text{sd} \left(\frac{\tilde{b}}{\sqrt{1-\tilde{b}^2}} Y_t \right)$	ρ	Av no of obs per group (N x T)
Above Trend June 1999 – September 2001	Coeff	na**	0.1123**	0.0125**	10,757
	SE		(0.0278)	(0.0061)	
	Chi Sq(1)			21.41**	
Recession October 2001- November 2003	Coeff	na**	0.0183	0.0003	24,344
	SE		(0.0182)	(0.0007)	
	ChiSq(1)			0.31	

* denotes significance at 5%; ** denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution.

Ref: \stata\p3p24.log

Table 4
Borrower dataset: Parameterisation of Equation 8

	Coeff	z-stat	Coeff	z-stat
Const	-2.626	-171.67	-3.870	-109.85
Income	-0.000	-0.030	0.000	0.84
(Base Age: 18-24)				
Age 25-29	-0.073	-3.82**	0.052	2.14**
Age 30-33	-0.103	-5.10**	-0.005	-0.20
Age 34-37	-0.078	-3.86**	0.028	1.08
Age 38-41	-0.138	-6.33**	-0.023	-0.84
Age 42-47	-0.112	-5.46**	0.015	0.59
Age 48-55	-0.200	-9.15**	-0.039	-1.42
Age 56 plus	-0.286	-11.78**	0.016	0.05
(Base: non self -employed)				
Self Employed	0.60	3.62**	0.052	2.51**
Time with bank	-0.001	-16.54**	-0.0004	-6.42**
No of cards	-0.008	-1.47	-0.010	-1.73
Balance/credit limit			1.809	66.90**
Δ Interest rate	-0.117	-1.89	0.123	1.55
Δ Unemployment	0.003	2.06**	0.002	1.23
Δ Real earnings	1.230	2.54**	0.427	0.70
Δ House price			-0.000	-1.92
Δ Total Credit			-0.000	-0.79
Sd(Y_t)	0.047	5.53**	0.050	5.22
ρ	0.0022	Chi sq 20.38**	0.0024	Ch sq (1)=15.85

* denotes significance at 5%; ** denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution

Ref: stata\p3p21.log & stata\p3p17.log

Table 5
Aggregate Level Dataset Parameterisations of Equation 6

	Const	$\text{sd} \left(\frac{\tilde{b}}{\sqrt{1 - \tilde{b}^2}} . Y_t \right)$	ρ
Coeff	-2.642**	0.1379**	0.0187**
SE	0.0164	0.0116	0.0031

No Groups (time periods) = 71

Av no of obs per group = 1

* denotes significance at 5%; ** denotes significance at 1%

The significance of the standard deviation is determined by dividing the coefficient by its standard error and assuming this has an approximate t-distribution.

Ref: \APACS\ a2.html

Table 6
Capital Requirements as a Multiple of LGD implied by Basel II Accord

PD	ρ	Capital Requirement
0.01	0.004	0.006373
0.01	0.006	0.008163
<i>0.01</i>	<i>0.04</i>	<i>0.030621</i>
0.02	0.004	0.011299
0.02	0.006	0.014391
<i>0.02</i>	<i>0.04</i>	<i>0.051418</i>
0.03	0.004	0.015635
0.03	0.006	0.019844
<i>0.03</i>	<i>0.04</i>	<i>0.068735</i>

Basel assumed parameters in italics.

Fig.1

